Math 136  Lesson 7/8.3.3 Practice Problems

1. In March 2007, 49% of Americans felt that the U.S. made the wrong decision in using military force in Iraq. A sample of 510 people taken in February 2008 showed that 275 now felt that the U.S. made the wrong decision in using military force in Iraq. (Data simulated from Pew Research Center for the People & the Press, 2008.) Do these data provide sufficient evidence to conclude that the proportion of Americans who felt that the U.S. made the wrong decision in using military force in Iraq rose between March 2007 and February 2008? Use a significance level of 0.05.

   a. Find \( n \) and \( x \).

   b. Calculate \( \hat{p} = \frac{x}{n} \).

   c. Determine \( H_0 \) and \( H_a \).

   d. Calculate \( \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \).

   e. Calculate the Test Statistic \( Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \).

   f. Find \( \alpha \) and calculate the \( P \)-value. Is the \( P \)-value < \( \alpha \)?

   g. State a conclusion.

   h. What does a Type I Error mean in this situation?

   i. What does a Type II Error mean in this situation?

   j. What type of error could your conclusion be in this situation?
2. A sample of 1500 Americans was asked how they paid their monthly bills. 810 of them paid bills by check (Clark, A., Funk, C. & Taylor, P., 2007). Do these data provide sufficient evidence at the 0.10 level of significance to conclude that the majority of Americans still paid their monthly bills by check in 2007?

a. Find \( n \) and \( x \).

b. Calculate \( \hat{p} = \frac{x}{n} \)

c. Determine \( H_0 \) and \( H_a \)

d. Calculate \( \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \)

e. Calculate the Test Statistic \( Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \)

f. Find \( \alpha \) and calculate the \( P \)-value. Is the \( P \)-value < \( \alpha \)?

g. State a conclusion.

h. What does a Type I Error mean in this situation?

i. What does a Type II Error mean in this situation?

j. What type of error could your conclusion be in this situation?
3. In 2013 the Gallup polling organization found that 45% of people of color view police officers as having high honesty and ethical standards. From a sample of 174 people of color taken in December 2014 only 40 of them still view police officers as having high honesty and ethical standards (Jones, J., 2014). Do these data provide significant evidence at the 0.01 level of significance to conclude that the proportion of people of color who view police officers as having high honesty and ethical standards dropped from 2013 to 2014?

a. Find $n$ and $x$.

b. Calculate $\hat{p} = \frac{x}{n}$

c. Determine $H_0$ and $H_a$

d. Calculate $\sigma_p = \sqrt{\frac{p(1-p)}{n}}$

e. Calculate the Test Statistic $Z = \frac{\hat{p} - p}{\sigma_p}$

f. Find $\alpha$ and calculate the $P$-value. Is the $P$-value $< \alpha$?

g. State a conclusion.

h. What does a Type I Error mean in this situation?

i. What does a Type II Error mean in this situation?

j. What type of error could your conclusion be in this situation?
4. The day that Pooh invented the game of Poohsticks, he played it by himself for quite a long time... "and when he went home for tea, he had won thirty-six and lost twenty-eight" (Milne, 1926, p. 94). Do these data provide sufficient evidence to conclude that Pooh loses only a minority of the time when playing Poohsticks by himself?

a. Find \( n \) and \( x \).

b. Calculate \( \hat{p} = \frac{x}{n} \)

c. Determine \( H_0 \) and \( H_a \).

d. Calculate \( \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \)

e. Calculate the Test Statistic \( Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \)

f. Find \( \alpha \) and calculate the \( P \)-value. Is the \( P \)-value < \( \alpha \) ?

g. State a conclusion.

h. What does a Type I Error mean in this situation?

i. What does a Type II Error mean in this situation?

j. What type of error could your conclusion be in this situation?
Answers

1. a.  $n = 510, x = 275$

   b.  $\hat{p} = \frac{x}{n} = \frac{275}{510} = 0.539$

   c.  $H_0 : p = 0.49 \ vs. \ H_1 : p > 0.49$

   d.  $\sigma_p = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.49)(0.51)}{510}} = 0.022$

   e.  Test Statistic:  $Z = \frac{\hat{p} - p}{\sigma_p} = \frac{0.539 - 0.49}{0.022} = 2.23$

   f.  $\alpha = 0.05, P\text{-value} = P(Z \geq 2.23) = 0.013$. Yes

   g.  Conclusion: Reject $H_0$. Conclude that the proportion of Americans who felt that the U.S. made the wrong decision in using military force in Iraq rose between March 2007 and February 2008.

   h.  Concluding that the proportion of Americans who felt that the U.S. made the wrong decision in using military force in Iraq rose between March 2007 and February 2008 when in fact it did not.

   i.  Concluding that the proportion of Americans who felt that the U.S. made the wrong decision in using military force in Iraq did not rise between March 2007 and February 2008 when in fact it did.

   j.  Type I

2. a.  $n = 1500, x = 810$

   b.  $\hat{p} = \frac{x}{n} = \frac{810}{1500} = 0.54$

   c.  $H_0 : p = 0.5 \ vs. \ H_1 : p > 0.5$

   d.  $\sigma_p = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.5)(0.5)}{1500}} = 0.013$

   e.  Test Statistic:  $Z = \frac{\hat{p} - p}{\sigma_p} = \frac{0.54 - 0.5}{0.013} = 3.08$

   f.  $\alpha = 0.05, P\text{-value} = P(Z \geq 3.08) = 0.001$. Yes

   g.  Conclusion: Reject $H_0$. The majority of Americans still paid their monthly bills by check in 2007.

   h.  Concluding that the majority of Americans still paid their monthly bills by check in 2007 when in fact they did not.

   i.  Concluding that the majority of Americans did not pay their monthly bills by check in 2007 when in fact they still did.

   j.  Type I
3. a. \( n = 174 \times 40 \)
   b. \( \hat{p} = \frac{x}{n} = \frac{40}{176} = 0.227 \)
   c. \( H_0 : p = 0.45 \) vs. \( H_a : p < 0.45 \)
   d. \( \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.45)(0.45)}{174}} = 0.038 \)
   e. Test Statistic: \( Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.227 - 0.45}{0.038} = -5.87 \)
   f. \( \alpha = 0.05, P\text{-value} = P(Z \leq -5.87) = 0.000 \) Yes
   g. Conclusion: Reject \( H_0 \). The proportion of people of color who view police officers as having high honesty and ethical standards dropped from 2013 to 2014
   h. Concluding that the proportion of people of color who view police officers as having high honesty and ethical standards dropped from 2013 to 2014 when in fact it did not.
   i. Concluding that the proportion of people of color who view police officers as having high honesty and ethical standards did not drop from 2013 to 2014 when in fact it did.
   j. Type I

4. a. \( n = 28 + 36 = 64, x = 28 \)
   b. \( \hat{p} = \frac{x}{n} = \frac{28}{64} = 0.438 \)
   c. \( H_0 : p = 0.5 \) vs. \( H_a : p < 0.5 \)
   d. \( \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.5)(0.5)}{64}} = 0.063 \)
   e. Test Statistic: \( Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.438 - 0.5}{0.063} = -0.98 \)
   f. \( \alpha = 0.05, P\text{-value} = P(Z \leq -0.98) = 0.164 \) No.
   g. Conclusion: Do not reject \( H_0 \). There is not enough evidence to conclude that Pooh loses only a minority of the time when playing Poohsticks by himself
   h. Concluding that Pooh loses only a minority of the time when playing Poohsticks by himself when in fact he does not.
   i. Concluding that Pooh does not lose only a minority of the time when playing Poohsticks by himself when in fact he does.
   j. Type II