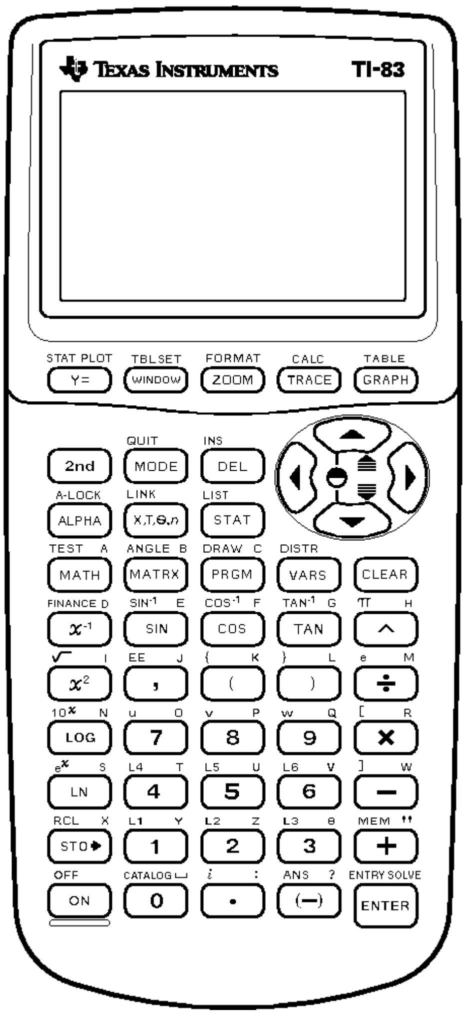
# Using the TI-83 Graphing Calculator
# in Math 111 at Tacoma Community College




**Version 1.3**

Scott MacDonald

This booklet is typeset using $\mathcal{A}\mathcal{M}\mathcal{S}$-LaTeX with the help of the `dvips` DVI to PS driver, Alladin Ghostscript, and GSView. These are all free programs.

All graphs (other than screenshots of the TI-83) are drawn using Graphmatica, and all graphics are converted to encapsulated postscript using WMF2EPS. These are shareware programs.

**Printing History:**

*Alpha Test Edition* December 1998 (No index, not proofread)
*Beta Test Edition* January 1999 (No index)
*Version 1.0* March 1999 (First complete edition)
*Version 1.1* May 1999 (Fixed a small problem with the index)
*Version 1.2* October 1999 (Fixed some typos and added material about the TI-83 Plus)
*Version 1.3* December 1999 (Some minor editing)

This printing: December 3, 1999.

# Contents

**Appendices**

# Preface

The Texas Instruments TI-83 graphing calculator is an extremely useful tool. The reason we recommend the TI-83 over many other calculators for Math 111 is that the TI-83 has a built in finance package and Time Value of Money (TVM) solver which, to my knowledge, no other calculator has. (This includes other TI calculators.)

Unfortunately, many students spend so much time trying to learn to use it that they have a difficult time learning both the calculator and the class material at the same time. This booklet is intended to aid in the use of the calculator by giving you a friendly keystroke guide to some of the more important functions.

## Notation

To illustrate keystrokes, I have a special notation which will be used throughout this booklet.

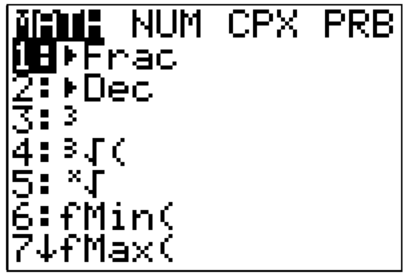
[TEXT] Indicates the primary function of the key. "TEXT" is printed in white on the key. [ON] indicates the "ON" button, for example.

[TEXT] Indicates the secondary function of the key. "TEXT" is printed in yellow above the key. This function is accessed by first pressing the yellow key at the top left of the main keypad (represented by [2nd]).

[TEXT] Indicates the alpha function of the key. "TEXT" is printed in green above the key. This function is accessed by first pressing the green key just below [2nd] (represented by [ALPHA]). Most alpha functions are just a single letter.

[#: TEXT] Indicates a option chosen from a menu. To select a menu item, either press the item's number or use the down arrow to highlight the number and press [ENTER].

**Example 0.1:**
The image below shows the MATH menu (gotten by pressing [MATH]).




To access the cube root function, for example, you would press [MATH] to bring up this menu, then either press [4] or press [▼] [▼] [▼] [ENTER]

**Example 0.2:**

**Keystrokes:**
[ON] [2nd] [A-LOCK] [H] [I] [2nd] [OFF]

If you perform the keystrokes above, you will turn the calculator on, write “HI” on the screen, and turn the calculator back off.

*Important: If you have done the above example, your calculator now shows "HI" in the upper left corner when you turn it on. This is meaningless to your calculator, so press [CLEAR] to get rid of it before you do anything else.*

**Example 0.3:**
This example illustrates what I mean by "option chosen from a menu".

**Keystrokes:**
[MATH] [4:³√(] [2] [7] [)] [ENTER]

This sequence evaluates the cube root of 27 and prints the answer (3) on the screen.

## Note to the Student

This booklet should be viewed as a work in progress. The best resource we have for knowing what topics should or should not be included is you, the student. If you have *any* suggestions concerning how this booklet could be improved please talk to your instructor or to the author (Scott MacDonald in Building 9, Room 23, (253)566-5236, smacdona@tcc.tacoma.ctc.edu). Welcome suggestions include but are not limited to: topics which should be added or deleted, sections which could be made clearer by being rewritten or reworded, and items which should be added to the index to make the manual easier to use.

# Chapter 1

# Introduction to the TI-83

## 1.1 The Basics

### Turning the Calculator On and Off

The button in the lower left hand corner of the keypad functions as both the "on" and the "off" button. "On" is the primary function of the button, while "off" is the secondary function.

**Keystrokes:**
To turn the calculator on, press ON.
To turn the calculator off, press 2nd OFF

### Controlling the Brightness of the Screen

Sometimes, especially if your batteries are low, the display on your screen may be so light that it is difficult or impossible to read. This is actually quite easy to fix.

**Keystrokes:**
To make the display darker, press 2nd, then press AND HOLD ▲ until the display is the desired darkness.
To make the display lighter, press 2nd, then press AND HOLD ▼.

### The Home Screen

The "home" screen is the screen on which most calculations are done. This is the screen that the calculator defaults to when you turn it on (unless it went off by itself, then it returns to the screen it was on when it went off). What screen you are using when you begin a keystroke sequence determines where the result is placed. If you are, for example, in the "Y=" screen (see Chapter 2) when you perform the keystrokes to compute an absolute value, then the function "ABS(" is placed in the definition of whatever function you were editing, and not on the home screen. It is thus quite important that you return to the home screen from wherever you are if you wish to perform calculations. To return to the home screen, you simply "quit" what you are doing.

**Keystrokes:**
To return to the home screen, press 2nd QUIT.

## Basic Calculations

Using the TI-83 for basic calculation is relatively straightforward. The TI-83 understands the order of operations, so you simply need to enter the expression as it appears on paper (provided that there are no fractions or exponents—if these are part of your calculation, then see sections 1.4 and 1.3). The ENTER key takes the place of an "=" sign.

**Example 1.1:**
To compute $(31.65 - 21.68)^2$, press
( 3 1 . 6 5 − 2 1 . 6 8
) $x^2$ ENTER

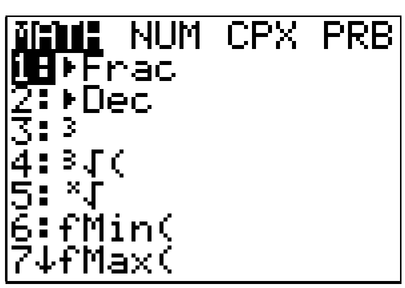
**CAUTION:** You MUST use the − key (below × and above +) for subtraction. The (-) key (to the left of ENTER) is the negative sign! The calculator interprets these two buttons differently!

The correct answer is 99.4009.

Notice the use of the $x^2$ key in the above example. This key is a special math function key. Note that it only enters the exponent $^2$ on the screen, not the entire expression $x^2$. Anytime you want to square something, enter the something, then press $x^2$.

## Finding Commonly Used Functions

Many commonly used math functions are the primary function of keys on the keypad, but many more are available using the MATH menu. To access this menu, press MATH. When you press this key, the menu shown below should appear on your screen.



The most commonly used functions from this menu are [1:▸Frac], which converts the entry to a fraction (if possible), [3:³], which cubes the entry (just like ($x^2$) squares the entry), and [4:∛(], which takes the cube root of an entry.

The absolute value function is under MATH/NUM and is [1:abs(]. (To get the MATH/NUM menu, press (MATH), then (▸).)

## 1.2 Using the Calculator's Memory

### Storing a Value for Later Use

It is often useful to store the result of a lengthy calculation so you can use the exact result without much effort at a later time. This process is not particularly difficult. Your calculator has many "memories" (at least 26–one for every letter in the alphabet). These are all accessed the same way.

**Keystrokes:**
To store the value 7.12976355 as "K", press
(7) (.) (1) (2) (9) (7) (6) (3) (5) (5) (STO▸) (ALPHA) [K]
(ENTER)

**Keystrokes:**
To store the result of the last calculation as "A" for later use, press
(STO▸) (ALPHA) [A]
At this point, your screen will show Ans→A. Press (ENTER), and the previous answer is stored in memory "A".

### Using a Stored Value

To use a value once it is stored, you simply need to replace the value in the expression with the letter you stored the value under.

**Keystrokes:**
If you have stored 7.12976355 as "K", then to compute $23 \cdot 7.12976355^2$, press
(2) (3) (×) (ALPHA) [K] ($x^2$) (ENTER)

(The answer should be 1169.17115).

## 1.3 Exponents

Entering exponents using the TI-83 is not difficult, but the process is not immediately obvious to someone who is not a mathematician or a computer person. The exponent key is (^). The reason for this symbol is that mathematics written on a single line (i.e., without superscripts) uses this notation to indicate an exponent. For example, to indicate "3·3", we usually write "$3^2$", but if we don't have the option of writing superscripts, then we write "3^2".

**Example 1.2:**
To compute $2^5$, press [2] [^] [5] [ENTER]. The correct answer is 32.

**Example 1.3:**
Then to compute $35B^4$, press [3] [5] [B] [^] [4] [ENTER].

The same process can be used for squaring (raising to the second power) and cubing (raising to the third power) things, too, but there are prettier methods for these very common powers. To square something, use the [$x^2$] key. To cube something, use the MATH menu option [3: ³].

**Keystrokes:**

- To square a number, enter the number then press [$x^2$] and press [ENTER].
- To cube a number, enter the number then press [MATH] [3: ³] and press [ENTER].
- To raise a number to any power, enter the number, press [^], then enter the exponent and press [ENTER].

If you want to raise 10 to a power, you can do that by either following the directions above, or by pressing [2nd] [$10^x$].

**Example 1.4:**
Suppose you want to compute $10^{-3}$.

Press [2nd] [$10^x$] [(-)] [3] [)]

Your calculator screen will show 10^(-3). Press [ENTER] to see that $10^{-3} = 0.001$.

The other number that can be raised to a power by use of a special key is the number $e$. $e$ is an irrational number which is *approximately* 2.718281828454590. It is used in Math 111 (and real life) in models for such things as continuously compounded interest, population growth, and radioactive decay. To raise $e$ to a power, you have two options: either press [2nd] [$e$] [^] and enter the exponent (using parentheses, if necessary—see section 1.4) or press [2nd] [$e^x$] and enter the exponent.

**Example 1.5:**
Find $e^{6.2}$. Round to 4 decimal places.
To do this, I will use the first method mentioned above.

Press [2nd] [e] [^] [(] [6] [×] [2] [)] [ENTER]. (The use of parentheses here is explained more in section 1.4.)

We can now see that $e^{6\cdot 2} \approx 162754.7914$. Please note that, despite the fact that these are all of the decimal places that the calculator shows, it is still an approximation!

**Example 1.6:**
Now do the same using the second method.

Press [2nd] [$e^x$] [6] [×] [2] [)] [ENTER]

Again, we find that $e^{6\cdot 2} \approx 162754.7914$.

## 1.4 Using Parentheses

The TI-83 is a great calculating machine, but it has its drawbacks. One of these drawbacks is that it cannot express mathematics exactly the way it appears on paper. For example, when a mathematician wishes to express the fraction "three-fifths", he or she writes "$\frac{3}{5}$". The TI-83 is incapable of writing a fraction in this way since all expressions are written on a single line. The way the calculator compensates is by writing fractions horizontally, say, "3/5" as opposed to the usual "$\frac{3}{5}$". This is fine as long as all you want to write is the fraction, but can be ambiguous. (Does the expression "1/5+x" mean "$\frac{1}{5+x}$" or "$\frac{1}{5}+x$"?)

Unfortunately, calculations such as the one above are all too common in real life. One formula that is commonly used is the formula for future value of an increasing annuity:

$$A = M\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}$$

Plug some numbers in for $M$, $r$, $m$, and $t$, and you have a calculation of exactly this type.

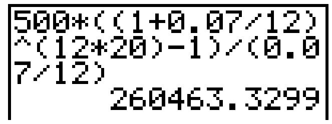
**Example 1.7:**
Evaluate the formula $A = M\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}$ for $M = 500$, $r = 0.07$, $m = 12$, and $t = 20$.
In normal notation, we write

$$A = 500\cdot\frac{\left(1+\frac{0.07}{12}\right)^{12\cdot 20}-1}{\frac{0.07}{12}},$$

but how do we get the calculator to compute this? The following keystrokes will do it:

**Keystrokes:**
[5] [0] [0] [×] [(] [(] [1] [+] [0] [.] [0]
[7] [÷] [1] [2] [)] [^] [(] [1] [2] [×] [2]
[0] [)] [−] [1] [)] [÷] [(] [0] [.] [0] [7]
[÷] [1] [2] [)] [ENTER]

Your screen will display the following:




Every one of the extra parentheses is NECESSARY. If even one gets left out, then your answer will not be correct!

Important things to note about the above example:

- There are parentheses which enclose the numerator of the fraction.
- There are parentheses which enclose the denominator of the fraction.
- There are parentheses which enclose the exponent.

The basic rules are as follows:

- If the numerator of a fraction consists of mathematical operations, enclose the numerator in parentheses. This is especially critical if the numerator includes addition or subtraction.
- If the denominator of a fraction consists of mathematical operations, enclose the denominator in parentheses. This is ALWAYS critical.
- If an exponent contains mathematical operations, enclose it in parentheses.
- If a root contains mathematical operations, enclose the radicand in parentheses.
- If an absolute value expression contains mathematical operations, enclose the expression in parentheses.
- IF YOU ARE NOT SURE WHETHER YOU SHOULD USE PARENTHESES OR NOT, USE THEM.

That last rule is important. The TI-83 doesn't care if you enter "$(3)/(5)$" or "$3/5$". These are the same. However, "$3/5*2$" is NOT the same as "$3/(5*2)$". (The first gives an answer of "$\frac{3}{5}\cdot 2 = 1.2$", while the second is "$\frac{3}{5\cdot 2} = 0.3$".)

Another frequently arising situation which requires the use of parentheses is the multiplication of a fraction and a variable. The expression "$\frac{3}{5}x$" MUST have parentheses if written on a single line. "$(3/5)x = \frac{3}{5}x$", while "$3/5x = \frac{3}{5x}$". The TI-83 and the TI-86 will allow you to get away with leaving the parentheses out in this situation, BUT DON'T. It is a bad habit to get into, and could come back to haunt you in the future. Besides, it makes your work hard to follow if someone else is looking at your screen or if you write your fractions horizontally. (This is improper notation, by the way, but if you MUST do it, then use parentheses as if you were entering information into your calculator.)

- When multiplying a fraction by a variable, you should enclose the fraction in parentheses. This is especially important if you are using any calculator other than the TI-83 or the TI-86.

# Chapter 2

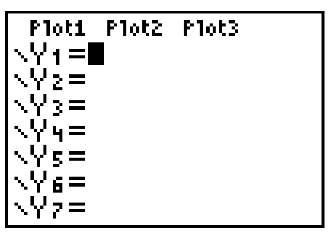
# Graphing

## 2.1 The Basics

In this class, we only use one of the TI-83's several graphing modes. The mode we use is the "Function" mode. This mode is the one that the TI-83 is in when you take it out of the package, so you should not have to explicitly set it. In case it is not in "Function" mode, press MODE ▼ ▼ ▼ ENTER 2nd QUIT.

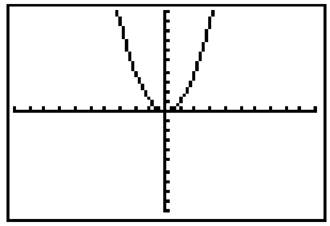
### Graphing One Function at a Time

Once in function mode, you will want to enter the function to be graphed. Press the Y= key. This brings up the screen shown below:




Enter the function as $Y_1$, then press GRAPH. When entering a function, the X,T,θ,n button is used to enter the variable. This variable is ALWAYS X on the calculator, regardless of what it is called in the problem.

**Example 2.1:**
Graph the function $f(x) = x^2$. Press Y= X,T,θ,n $x^2$ GRAPH. The resulting graph should look like: (if it doesn't, press ZOOM and select 6:ZStandard)

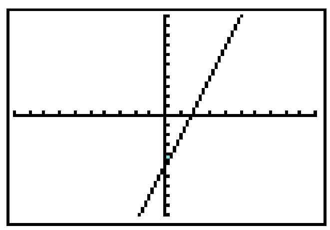


When you are done graphing a particular function, go to that function in the function entry screen and press [CLEAR].

**Example 2.2:**
Graph the function $q = 3p - 5$.

To do this, we must first recognize that $q$ is a function of $p$. In other words, $q$ in this function is playing the role that $y$ usually does, while $p$ is playing the role of $x$.

Press [Y=]. If there is anything entered as $Y_1$, press [CLEAR]. Now we enter the function by pressing [3] [X,T,θ,n] [−] [5]. Now graph the function by pressing [GRAPH]. The graph below should result.



## Graphing Two or More Functions at Once

It is often useful to be able to graph more than one function on the screen at the same time. Some possible reasons for this are to compare the shapes of different graphs, to see which function increases faster, or to see where the graphs intersect. The need to graph more than one function at a time is the reason that the function entry screen allows you to enter up to 10 functions (only 7 show at a time, but if you scroll down by holding down the [▼] button, then you can see functions numbered 8, 9 and 0). Generally, you won't graph more than two or three at a time, as more makes the screen pretty cluttered.

To graph more than one function, simply enter the first function as $Y_1$, the second as $Y_2$, and so on. When you are done entering functions, press [GRAPH], and the calculator will draw $Y_1$ followed by $Y_2$, etc.

## Graphing a Piecewise-Defined Function

There are really two ways to graph a piecewise-defined function. Both methods, however, require that you first take the calculator out of the default "connected" mode. To do so, press MODE ▼ ▼ ▼ ▼ ▶ ENTER. This will cause "Connected" to be un-highlighted and "Dot" to be highlighted. Your calculator will now not connect all the points it draws when graphing a function. Press 2nd QUIT to return to the home screen.

**Method 1:** The first method of graphing a piece-wise defined function is to enter each piece (along with it's restriction) as a separate function. The inequalities $>$, $<$, $\geq$, and $\leq$ are found in the TEST menu. This is accessed by pressing 2nd TEST. (TEST is the second function of MATH).

**Example 2.3:**

Graph the function $f(x) = \begin{cases} x^2 & \text{if } -3 < x \leq 1 \\ x+1 & \text{if } x > 1 \end{cases}$

To do this, first put the calculator in "dot" mode (see above), then press Y= and enter $x^2(-3 < x)(x \leq 1)$ for $Y_1$ as follows:

X,T,θ,n $x^2$ ( (-) 3 2nd TEST 5:< X,T,θ,n ) ( X,T,θ,n 2nd TEST 6:≤ 1 )

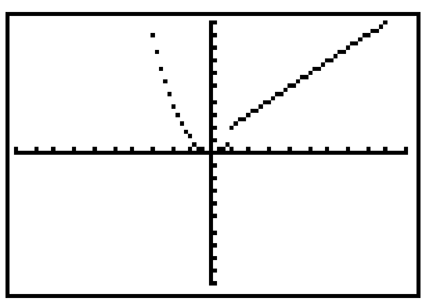
NOTE THAT THE RESTRICTION IS BROKEN INTO TWO PIECES. This will not work if you try to enter $-3 < x \leq 1$ all at once.

Now move to $Y_2$ and enter $(x+1)(x > 1)$ by pressing

( X,T,θ,n + 1 ) ( X,T,θ,n 2nd TEST 3:> )

NOTE THE PARENTHESES AROUND "$x+1$"! Without them, this does not work.

Press GRAPH, and you should get the graph shown below:



The dotted aspect of the parabola (left part of the graph) is because the calculator is in "dot" mode. In reality, all of those points should be connected with a smooth curve.

**Method 2:** Method 2 differs from method 1 in that the function is not entered in pieces. Instead, it is entered as a single function defined as the sum of all of the pieces (including restrictions). This is not phrased very well, so see the following example.

**Example 2.4:**
To obtain the same graph as in the previous example, put the calculator in "dot" mode, press [Y=] and enter the following as $\mathsf{Y}_1$:

[X,T,θ,n] [$x^2$] [(] [(-)] [3] [2nd] [TEST] [5:<] [X,T,θ,n] [)] [(] [X,T,θ,n] [2nd] [TEST] [6:≤] [1] [)] [+] [(] [X,T,θ,n] [+] [1] [)] [(] [X,T,θ,n] [2nd] [TEST] [3:>] [1] [)]

This looks pretty nasty, but it simply enters $\mathsf{Y}_1 = x^2(-3 < x)(x \leq 1) + (x + 1)(x > 1)$. When you press [GRAPH], you should get the same graph as before.

NOTE: When you are done graphing piece-wise defined functions, return the calculator to "connected" mode by pressing [MODE] [▼] [▼] [▼] [▼] [ENTER].

## Deselecting a Function for Graphing

On occasion, you may wish to have a function entered in the function list, but you don't want to graph that function. Most often this occurs when you know that you'll need to graph the function later, but you don't need the graph now and you don't want to have to re-enter it when you need it. In this case, you will want to "deselect" the function and then "reselect" it when you want to see the graph.

**Keystrokes:**
To "deselect" a function, move to that function in the function list by using [▼] and/or [▲], then press [◄] to move the cursor onto the "=" and press [ENTER] to deselect. The "=" indicating that the function will not graph when the [GRAPH] button is pressed.

**Keystrokes:**
To "reselect" a deselected function, perform the same keystrokes as above until the "=" is highlighted.

## TRACEing a Graph

When you have the graph(s) of one or more functions displayed on the screen, you can use the TRACE function to move along the graph and see $x$ and $y$ values at various points on the graph. To do so, press the [TRACE] button. A crosshair appears on one of the graphs (the equation for the current graph appears in the upper left corner of the graph window). The bottom left corner contains the current $x$-coordinate and the center of the bottom contains the current $y$-coordinate. Use the [◄] and [►] buttons to move along the current graph. If you have more than one active graph, then you can use the [▲] and [▼] buttons to change graphs.

## 2.2 Parentheses and Graphing

The use of parentheses when entering functions for graphing follows the same basic rules as for calculations (see section 1.4):

- If the numerator of a fraction consists of mathematical operations, enclose the numerator in parentheses. This is especially critical if the numerator includes addition or subtraction.
- If the denominator of a fraction consists of mathematical operations, enclose the denominator in parentheses. This is always critical, even if the numbers on the bottom are to be multiplied.
- If an exponent contains mathematical operations, enclose it in parentheses.
- If a root contains mathematical operations, enclose the radicand in parentheses.
- If an absolute value expression contains mathematical operations, enclose the expression in parentheses.
- When multiplying a fraction by a variable, you should enclose the fraction in parentheses. This is especially important if you are using any calculator other than the TI-83 or the TI-86.
- IF YOU ARE NOT SURE WHETHER YOU SHOULD USE PARENTHESES OR NOT, USE THEM.

**Example 2.5:**

Suppose you wanted to graph the function $f(x) = \frac{x+3}{5-x}$. You could try entering it into the calculator using the keystrokes

[X,T,θ,n] [+] [3] [÷] [5] [–] [X,T,θ,n]

This results in the screen display X + 3/5 – X, which, in normal notation, is $x + \frac{3}{5} - x$. This is bad.

What we need is some way to tell the TI-83 that the entire $x+3$ is on top of the fraction and that the entire $5-x$ is on the bottom. The answer is parentheses. Instead of the keystrokes above, type in

[(] [X,T,θ,n] [+] [3] [)] [÷] [(] [5] [–] [X,T,θ,n] [)]

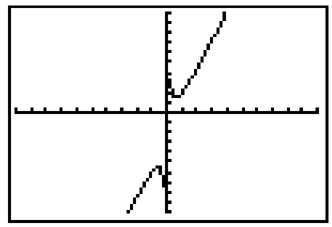
The screen display this time is (X + 3)/(5 – X), which is what we wanted.

If you omit the parentheses which enclose the top and the bottom of a fraction, you are NOT going to get the output you desire. Compare the two graphs in example 2.6, below.

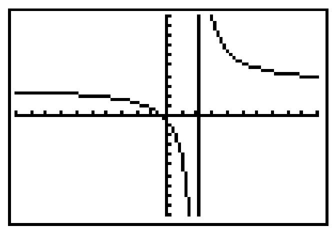
**Example 2.6:**

A homework exercise asks students to graph the function $f(x) = \frac{3x+1}{x-2}$ on their calculators. Student A enters the function

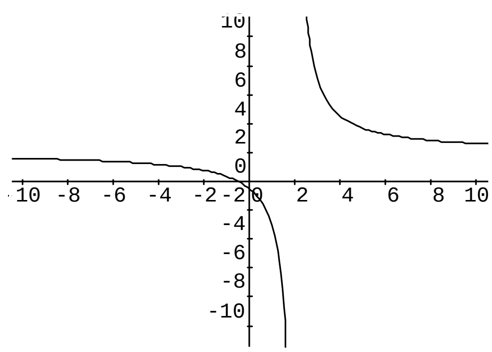
$\mathsf{Y}_1 = 3\mathsf{X} + 1/\mathsf{X} - 2$
His graph looks like



Student B enters the function
$\mathsf{Y}_1 = (3\mathsf{X} + 1)/(\mathsf{X} - 2)$
His graph looks like



The true graph looks like




Why is the first graph wrong? Since the student forgot to put parentheses around the numerator and around the denominator of the fraction, what he really graphed was the function $g(x) = 3x + \frac{1}{x} - 2$. As you can see, the graph is really quite different.

**Example 2.7:**
Graph the function $y = \frac{2}{3}x - 5$.

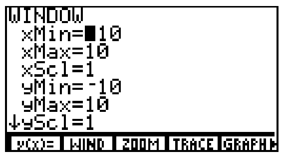
If we didn't think about it, we might begin by pressing [Y=] and entering the function as [2] [÷] [3] [X,T,θ,n] [−] [5]. While the TI-83 and TI-86 will allow you to get away with this sloppy method, most other calculators will not—so avoid it.

Instead, put parentheses around the fraction: ( 2 ÷ 3 ) X,T,θ,n − 5. The resulting graph should be the same regardless of what calculator you are using.

## 2.3 Adjusting the Field of View

### The WINDOW Screen

Press the WINDOW key. The screen displayed should look like

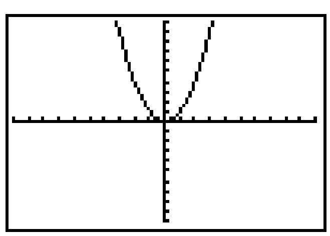



If your screen doesn't look like this, try pressing ZOOM, selecting 6:ZStandard, and pressing WINDOW again.

This first three lines of the screen (after WINDOW) are telling you that the left side of the graphing screen is at $x = -10$, the right side is at $x = 10$, and the tick marks on the $x$-axis occur every one unit. The next three lines tell you that the bottom of the graphing screen is at $y = -10$, the top is at $y = 10$, and the tick marks on the $y$-axis occur every one unit.

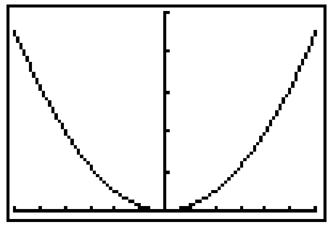
**NOTATION:** From here on out, I will use the notation X:[Xmin,Xmax;Xscl] and Y:[Ymin,Ymax;Yscl] to talk about the entries on this screen. The entries for the standard window shown above, then, are X:[-10,10;1] and Y:[-10,10;1].

**Example 2.8:**
Graph the function $y = x^2$ on the standard window (i.e., don't change any of the settings shown above). You should get the graph shown below.



Press WINDOW, change the entries to X:[-3,3;1] and Y:[0,10;1], and press GRAPH. Note the change in the graph. The $x$-axis now only goes from $-3$ to 3 and the $y$-axis from 0 to 10. Notice how many tick marks are on each axis now (6 on the $x$-axis and 10 on the $y$-axis).

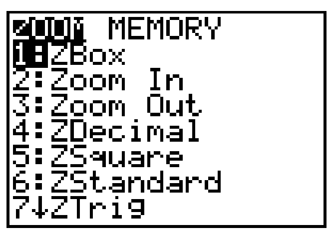
When you are done with the previous view, press WINDOW again and change the entries to X:[-3,3;.5] and Y:[0,10;2]. Press GRAPH. Now there are 12 tick marks on the *x*-axis (at −3, −2.5, −2, −1.5, −1, −0.5, 0.5, 1, 1.5, 2, 2.5, and 3—the marks at −0.5 and 0.5 are mostly hidden by the graph). There are 5 tick marks on the *y*-axis (at 2, 4, 6, 8, and 10).



It is a good idea when you are done to always return the window to the standard settings X:[-10,10;1] and Y:[-10,10;1]. That way you are always starting in the same place.

## The ZOOM Menu

Sometimes there are features of a graph that you would like a closer look at, but you don't know exactly what values to enter in the WINDOW screen to get the desired results. This is where the functions of the ZOOM menu can come in handy. Press the ZOOM key. The menu displayed should look like



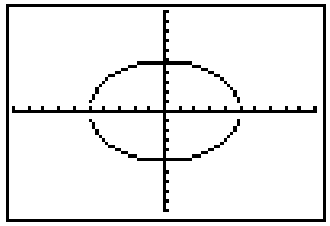

The arrow in place of the colon for option 7 tells you that there are more options. Press and hold ▼ to see the last three options (numbered 8, 9, and 0).

You won't use most of the options on this menu in Math 111, but there are a few that you may find useful:

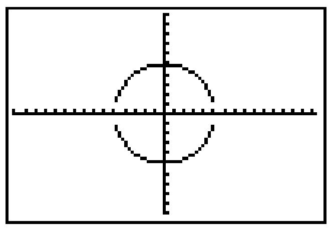
1:ZBox You can use this option to draw a rectangular box around a part of the graph that you want a closer view of. After selecting this option, use the arrow keys to move to the top left corner of the box you want and press ENTER, then use the arrow keys to move to the bottom right corner of the box you want and press ENTER.

2:Zoom In This option allows you to zoom in on the graph at a particular point. After selecting this option, use the arrow keys to move the cursor to the point on the graph that you would like to be the center of the new, zoomed graph and press ENTER.

5:ZSquare You may have noticed that the screen is wider than it is tall. Due to this and other problems caused by using 1:ZBox, graphs can become very distorted. A circle* graphed on the standard viewing window looks like this:



The fact that it doesn't touch the *x*-axis is troubling, but not the point. The point is that it looks like an ellipse because it is longer than it is tall. The 5:ZSquare option can save the day. It "squares" the screen so that the units are the same length in both the *x* and the *y* directions. The result of selecting 5:ZSquare on this graph is:



The graph still doesn't touch the *x*-axis, but we can't do anything about that. At least the shape is more circle-like now.

6:ZStandard This option returns the view to the standard viewing window (X:[-10,10;1] and Y:[-10,10;1]).

0:ZoomFit This option is intended for use when you don't know what values to enter for Ymin and Ymax. If you select this option, the calculator accesses the Xmin and Xmax as displayed on the WINDOW screen, then computes Ymin and Ymax to display as much of the graph of each active function as possible. Note: 0:ZoomFit does not change the Yscl setting, so you may need to do so by hand in order to have a meaningful *y*-axis. THIS OPTION SHOULD ONLY BE USED IF YOU KNOW THE BASIC SHAPE OF THE GRAPH(S) IN QUESTION. If you don't know the basic shape, then it is difficult to know if the result is actually useful or not.

## 2.4 Advanced Graphing Techniques

### Evaluating a Function

One drawback of TRACE that you will discover very quickly is that you cannot control exactly what values of *x* it goes to. The values it displays are the ones that it just happened

*The circle is graphed by using the equations $Y_1 = \sqrt{(25 - X^2)}$ and $Y_2 = -\sqrt{(25 - X^2)}$.

to compute in the process of drawing the graph. With rare exception, these values are not the ones you want. It is, however, not a difficult task to find the value of a function. There are two methods:

### Using TRACE

Use the calculator to draw the graph of the function in question so that the value of $x$ that you want to use is between Xmin and Xmax. Press TRACE and then enter the value of $x$.

**Example 2.9:**
Suppose you wish to evaluate $F(x) = 2x^3 - 3x^2 + x - 1$ for $x = -4$.

Since $-4$ is between $-10$ and 10, the standard window (see section 2.3) will do. Graph this function, press TRACE (-) 4 ENTER. The bottom of the screen reads X = -4 Y = -181, meaning that $F(-4) = -181$.

### Without Using TRACE

The second method is more algebraic in nature and uses function notation. Enter the function you wish to evaluate as one of the functions in the function entry screen and return to the home screen. Then use the VARS/Y-VARS/Function menu to choose the appropriate function and evaluate it using the $f(x)$ notation.

**Example 2.10:**
Evaluate $G(t) = 4t^2 - 2t + 1$ for $t = 15$.

Press Y= and enter the function (with X instead of $t$, of course) as $Y_1$, then press 2nd QUIT to return to the home screen.

Now press VARS ▶ (to move to the Y-VARS menu), select 1:Function..., then select 1:Y₁. This puts $Y_1$ on the screen.

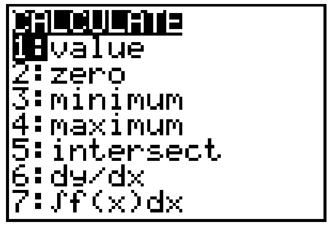
Next, press ( 1 5 ) and press ENTER.

We now see that $G(15) = 871$.

*NOTE: This problem can also be done using the method shown in the last example, but you would have to adjust the viewing window first so that 15 lies between* Xmin *and* Xmax*. See section 2.3 for how to do this.*

## The CALCULATE Menu

Press 2nd CALC (located on the TRACE key). This brings up the following menu:




This is another menu that you will not use a lot of in Math 111, but some of the options are extremely useful.

[2:zero] This option can be used to find the $x$-intercepts of a graph. When you choose this option, the graph is redisplayed with the question "Left Bound?" At this point, you should enter an $x$ value which is a little bit less than the suspected $x$-intercept. The calculator then asks "Right Bound?" Enter an $x$ value which is a little bit greater than the suspected $x$-intercept. Finally, the calculator asks "Guess?". Enter a value *between those entered for "Left Bound" and "Right Bound"*.

**Example 2.11:**
Graph the function $y = x^3 - x - 1$ on the window X:[-5,5;1] and Y:[-5,5;1]. There is an $x$-intercept between 1 and 2. We will use the [2:zero] function to find this $x$-intercept.

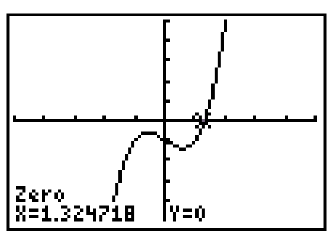
Press [2nd] [CALC] and select [2:zero].

When the calculator asks, "Left Bound?", press [1] to tell the calculator that the intercept is greater than 1.

When the calculator asks, "Right Bound?", press [2] tell the calculator that the intercept is less than 2.

When the calculator asks, "Guess?", press [1] [.] [3] to guess 1.3.

The calculator gives the following display, showing that the $x$-intercept occurs when $x \approx 1.324718$.




[3:minimum] If a function has a local minimum value between Xmin and Xmax, then this option can be used to find the minimum value.

**Example 2.12:**
Graph the function $y = x^3 - x - 1$ on the window X:[-5,5;1] and Y:[-5,5;1]. There is a local minimum between 0 and 1, so we will use the [3:minimum]

function to find it.

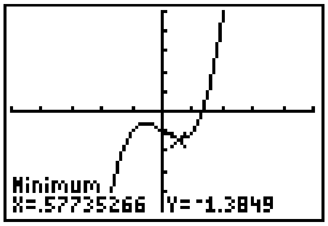
Press 2nd CALC and select 3:minimum.

When the calculator asks, "Left Bound?", press 0 to tell the calculator that the minimum occurs at a value of $x$ greater than 0.

When the calculator asks, "Right Bound?", press 1 to tell the calculator that the minimum occurs at a value of $x$ less than 1.

When the calculator asks, "Guess?", press . 5 to guess that the $x$-coordinate at the minimum is 0.5.

The calculator gives the following display, showing that the minimum value is $y \approx -1.3849$ and occurs when $x \approx 0.57735266$.




4:maximum Just like 3:minimum, except that it finds a local maximum value if one exists.

5:intersect This option can be used to find the intersection point(s) of two graphs.

**Example 2.13:**
Graph the functions $f(x) = x^3 - x - 1$ and $g(x) = 2 - x$ on the window X:[-5,5;1] and Y:[-5,5;1]. These graphs intersect in the first quadrant. We will use the 5:intersect function to find the point of intersection.

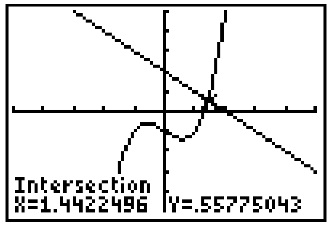
Press 2nd CALC and select 5:intersect.

The calculator asks the question, "First Curve?" Answer this question by using ▲ and/or ▼ to move onto the graph of $f$ (the formula will appear in the upper left corner of the window) and pressing ENTER.

The calculator then asks the question, "Second Curve?" Answer by using ▲ and/or ▼ to move onto the graph of $g$ and pressing ENTER.

The calculator now asks for a guess. It looks like the $x$-coordinate of the point of intersection is near $x = 1.5$, so enter 1 . 5 and press ENTER.

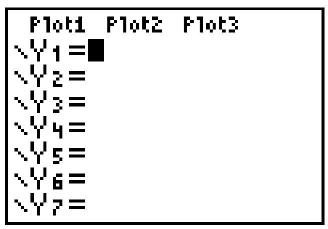
The calculator displays the following, indicating that the intersection point is at $x \approx 1.4422496$ and $y \approx 0.55775043$.




## Graphing Linear Inequalities

**NOTE:** Shading in this section is according to the convention in the text **Finite Mathematics & Calculus Applied to the Real World**, by Stefan Waner and Steven R. Costenoble. (This is the text used in Math 111 and Math 112 at TCC.)

Bring up the function entry screen by pressing [Y=].




The before each function is actually an indication of the current drawing mode for the function. This can be changed by pressing [◄] [◄] to move the cursor to the and pressing [ENTER]. Press [ENTER] repeatedly to cycle through the entire list. The interesting possibilities are:

**(or *connected*)** In this mode, all graphs are drawn with all plotted points connected with a relatively smooth curve. This is the default mode, and if you press clear to delete a function, it will automatically go back to this mode.

**(or *thick lines*)** In this mode, the lines connecting the plotted points are thicker. This mode is usually used to help tell two graphs apart, or see if one is drawn directly on top of the other.

**(or *shade above*)** In this mode, the graph is drawn as usual, but shading extending from the curve to the top of the screen is also drawn in.

**(or *shade below*)** Same as *shade above*, except that the shading is below the curve.

**(or *dotted*)** In this mode, the points plotted by the calculator are not connected. It is impossible to see the difference between this and *connected* mode unless the graph increases or decreases rapidly at some point. (Such as $f(x) = \frac{1}{1+x}$.)

The other two options can be ignored for now.

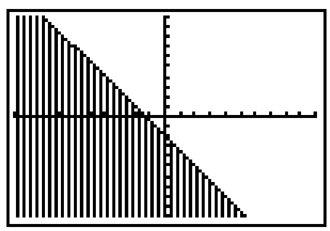
How does this all apply to linear inequalities? For starters, when shading a linear inequality (assuming the line is NOT vertical), it is only necessary to determine whether one should shade above the line or below the line. To do this, put the linear inequality in slope-intercept form. If the inequality says $y > \dots$ or $y \geq \dots$, then shade below. If the inequality says $y < \dots$ or $y \leq \dots$, then shade above.

**Example 2.14:**
Graph the linear inequality $3x - 2y \leq 4$.

Solving for $y$, we get $y \geq -\frac{3}{2}x - 2$. (Notice that the inequality reversed direction because I have to divide by $-2$.) This tells me that I want to shade below the line.

Press [Y=] and enter -(3/2)X − 2 under $Y_1$. Next, press [◄] until the cursor is blinking under the symbol at the far left side of the screen next to $Y_1$ and press [ENTER] until ◣ shows. (If you accidentally go past it, keep pressing [ENTER] and it will come around again.) Now press [GRAPH].

If you are graphing on the standard window, you should get the following result:



The difficulty in graphing systems of inequalities, then, is that many such systems include an inequality of the form $x \geq a$ or $x \leq a$. This is a problem because the boundary line is vertical and when in function mode the TI-83 cannot graph vertical lines. We will not change modes. Instead, we will fool the calculator by graphing a VERY steep line that looks, for all intents, like a vertical line. In this way, we can "cheat" and get a reasonable approximation of the graph of the system.

**Example 2.15:**
Graph the linear inequality $x \geq 2$.

Do this by entering -100000(X − 2) and shading below.

In general, you can graph an inequality of these forms by graphing $y = -100,000(x-a)$ and shading below if the inequality is "$\geq$" and shading above if the inequality is "$\leq$".

**Example 2.16:**
*This is example 6 on page 248 of the class text*
Sketch the region of solutions of the following system of inequalities, and list

the coordinates of all the corner points.

$$3x - 2y \leq 6$$
$$x + y \geq -5$$
$$y \leq 4$$

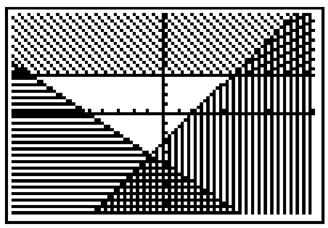
Solving for $y$ in each of the inequalities above, we get

$$y \geq \frac{3}{2}x - 3$$
$$y \geq -x - 5$$
$$y \leq 4$$

Thus, we enter these into the calculator as shown below:

```
Y1=(3/2)X-3
Y2=-X-5
Y3=4
```

Press GRAPH and you should get



We now find the points of intersection by using the CALCULATE/intersect function:

Press 2nd CALC and select 5:intersect.

To find the point at the bottom of the triangle, select $Y_1 = (3/2)X - 3$ for the first curve and select $Y_2 = \text{-}X - 5$ for the second. For the guess, enter $-1$. We find that the bottom point of the triangle is at $x = -0.8$ or $-\frac{4}{5}$ and $y = -4.2$ or $-\frac{21}{5}$. So the point of intersection is $\left(-\frac{4}{5}, -\frac{21}{5}\right)$

We will now find the point at the upper left corner. Press 2nd CALC and select 5:intersect again. For the first curve, press ▼ to select $Y_2 = \text{-}X - 5$. For the second, select $Y_3 = 4$. For the guess, enter $-8$. We find that the intersection is at $(-9, 4)$.

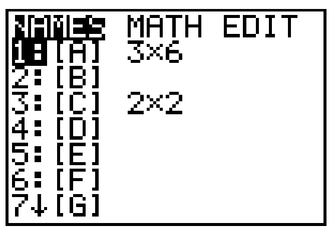
Now we find the point at the upper right corner. Press 2nd CALC and select 5:intersect once again. For the first curve, select $Y_1 = (3/2)X - 3$ and for the second, press ▼ to select $Y_3 = 4$. For the guess, enter 4. The calculator shows $X = 4.6666667$ $Y = 4$, meaning that the point of intersection is at $\left(\frac{14}{3}, 4\right)$.

# Chapter 3

# Matrices

## 3.1 Entering and/or Editing a Matrix

Press MATRX. The screen displayed will look a lot (although not necessarily exactly) like the one below:




(Your calculator may not have the "3×6" or "2×2" entries. Or you might have different entries altogether in that column. That's okay.) Notice that there are three submenus: NAMES, MATH, and EDIT. To enter or edit a matrix, you must select that matrix from the MATRX/EDIT menu.

To edit matrix [A], for example, press MATRX ▶ ▶ and select 1:[A]. If this is the first time you have entered a matrix as [A], then your screen will show

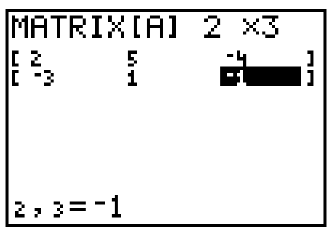

If a matrix has been entered on your calculator as [A] before, then the display will show that matrix instead. The display above indicates that [A] is a 1x1 matrix whose only entry is "0". To change [A] to a 2×3 matrix, press 2 ENTER 3 ENTER. The screen changes to show


The upper right corner now indicates a 2×3 matrix, while the main part of the screen shows a picture of the matrix. (If the matrix is too wide for the screen, then it will simply run off the edge of the screen and will scroll to show other entries if you press [►] and/or [◄].) The bottom left corner of the screen indicates that the cursor is on the first row, first column, and the entry there is 0.

Enter the matrix $\begin{bmatrix} 2 & 5 & 4 \\ -3 & 1 & -1 \end{bmatrix}$ by pressing

[2] [ENTER] [5] [ENTER] [4] [ENTER]
[(-)] [3] [ENTER] [1] [ENTER] [(-)] [1] [ENTER].

Your screen should look like




Note that the bottom left corner now indicates that the cursor is in row 2, column 3, and the entry there is −1.

You are now done entering this matrix. Press [2nd] [QUIT] to return to the home screen.

Suppose that instead of $\begin{bmatrix} 2 & 5 & 4 \\ -3 & 1 & -1 \end{bmatrix}$, you meant to enter $\begin{bmatrix} 2 & 5 & 4 \\ -3 & \mathbf{4} & -1 \end{bmatrix}$. This is, luckily, easy to fix.

Press [MATRX] [►] [►] and select [1:[A]], then use the arrow keys to move to the center of the second row (it reads 1 and we want to change it to 4). Press [4] [ENTER] and the repair is made.

Now let's add a row to the matrix. Instead of $\begin{bmatrix} 2 & 5 & 4 \\ -3 & 4 & -1 \end{bmatrix}$, suppose we want $\begin{bmatrix} 2 & 5 & 4 \\ -3 & 4 & -1 \\ -2 & 0 & 10 \end{bmatrix}$.

To enter the new row, press [MATRX] [►] [►] and select [1:[A]]. Press [3] [ENTER], and the calculator adds a third row. Use the arrow keys to move to the first entry in the third row and press [(-)] [2] [ENTER] [0] [ENTER] [1] [0] [ENTER].

Adding a new column to a matrix works in exactly the same was as adding a new row, except that you change the number of columns instead. All entries in the new column will be "0" until you change them.

## 3.2 Matrix Algebra

First let me make it clear that by "Matrix Algebra", I mean matrix addition, subtraction, and multiplication, scalar multiplication, and matrix inversion. I do NOT mean row operations. Row operations are discussed in section 3.3.

**Important Note:** *None of the algebraic operations store the result in the calculator. In other words, if you have a matrix $A$ stored in the calculator and you want to have the matrix $2A$ instead, you will have to store the results yourself. In this case, just press* [2] [MATRX], select [1: [A]], press [STO▶] [MATRX], select [1: [A]], and press [ENTER]. This will replace the old matrix $A$ with the new matrix $2A$.

The only trick to doing matrix algebra is to remember that any matrix you wish to use must be chosen from the MATRX/NAMES menu.

**Example 3.1:**
Let $A = \begin{bmatrix} 2 & 4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 & -15 \\ 3 & -2 & -5 \end{bmatrix}$. Find $A+B$, $A-B$, and $2A-5B$.

Enter the matrices $A = \begin{bmatrix} 2 & 4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 & -15 \\ 3 & -2 & -5 \end{bmatrix}$ into your calculator as matrix [A] and matrix [B], respectively. (See section 3.1 for how to do this.) When you are done, press [2nd] [QUIT] to return to the home screen.

We start by computing $A+B$. Press [MATRX], select [1: [A]], press [+] [MATRX], select [2: [B]], and press [ENTER]. You should get

$$A+B = \begin{bmatrix} 0 & 5 & -10 \\ 4 & 0 & -2 \end{bmatrix}$$

Now we compute $A-B$. Press [MATRX], select [1: [A]], press [–] [MATRX], select [2: [B]], and press [ENTER]. You should get

$$A-B = \begin{bmatrix} 4 & 3 & 20 \\ -2 & 4 & 8 \end{bmatrix}$$

Finally, we compute $2A-5B$. Press [2] [MATRX], select [1: [A]], press [–] [5] [MATRX], select [2: [B]], and press [ENTER]. You should get

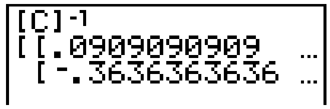
$$2A-5B = \begin{bmatrix} 14 & 3 & 85 \\ -13 & 14 & 31 \end{bmatrix}$$

As you can see in the last part of that example, even scalar multiplication is pretty easy. The next example illustrates how easy it is to invert a matrix using your calculator.

**Example 3.2:**

Let $C = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$. Enter this as matrix [C] and return to the home screen.

Press MATRX, select 3: [C], and press $x^{-1}$ ENTER. The screen displays




The dots (...) at the right hand edge of the screen indicate that the matrix runs off the edge of the screen. Use the ▶ and ◀ keys to view the entire matrix. Your calculator should give you the answer

$$C^{-1} = \begin{bmatrix} .0909090909 & .1818181818 \\ -.3636363636 & .2727272727 \end{bmatrix}$$

This, unfortunately, is not really the way we like to see this type of answer. We would rather see fractions. Press MATH, select 1: ▶Frac, and press ENTER. The answer in fraction form is

$$C^{-1} = \begin{bmatrix} \frac{1}{11} & \frac{2}{11} \\ -\frac{4}{11} & \frac{3}{11} \end{bmatrix}$$

**Example 3.3:**

If $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$, find $AB$.

Enter $A$ and $B$ into your calculator as [A] and [B], respectively, then press 2nd QUIT to return to the home screen.

Press MATRX, select 1: [A], press MATRX, select 2: [B], and press ENTER. The answer you should get is

$$AB = \begin{bmatrix} 14 & 18 \\ 34 & 42 \end{bmatrix}$$

## 3.3 Performing Transpositions and Row Operations

The MATRX/MATH menu (see below) allows access to many operations, including a function for finding the transpose of a matrix and functions for performing row operations.

## Transpositions

The menu option [2:ᵀ] is used to take the transpose of a matrix.

**Example 3.4:**
Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$. Find $A^T$.

Enter $A$ as matrix [A], return to the home screen, and press (MATRX), select [1:[A]], press (MATRX) (▶), select [2:ᵀ], and press (ENTER).
The answer returned by your calculator should be

$$A^T = \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$$

## Row Operations

The row operations are options A-F of the MATRX/MATH menu.

[A:ref(] Syntax: ref(*matrix*). Puts the specified matrix in row echelon form.

**Example 3.5:**
Assume the matrix $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$ is stored as [A].

To put this matrix in row echelon form, press (MATRX) (▶), select [A:ref(], press (MATRX), select [1:[A]], and press ( ) ) (ENTER).

The result displayed on your screen should be (you have to scroll right to see it all) $\begin{bmatrix} 1 & 1.142857143 & 1.285714286 \\ 0 & 1 & -2.454545455 \\ 0 & 0 & 1 \end{bmatrix}$

[B:rref(] Syntax: rref(*matrix*). Puts the specified matrix in reduced row echelon form. See the example above for use of this syntax.

[C:rowSwap(] Syntax: rowSwap(*matrix,rowA,rowB*). Swaps *rowA* and *rowB* in the specified matrix.

**Example 3.6:**
Assume the matrix $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$ is stored as [A].

To swap rows two and three, press (MATRX) (▶), select [C:rowSwap(], press

MATRX, select 1:[A], and press , 2 , 3 ) ENTER.

The result displayed on your screen should be

$$\begin{bmatrix} 2 & 3 & -1 \\ 7 & 8 & 9 \\ 3 & 5 & 0 \end{bmatrix}$$

D:row+( Syntax: row+(*matrix,rowA,rowB*). Adds *rowA* and *rowB* in the specified matrix and replaces *rowB* with the result.

**Example 3.7:**

Assume the matrix $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$ is stored as [A].

To add rows 2 and 3 and store the result in row 2, press MATRX ▶, select D:row+(, press MATRX, select 1:[A], and press , 3 , 2 ) ENTER.

The result displayed on your screen should be

$$\begin{bmatrix} 2 & 3 & -1 \\ 10 & 13 & 9 \\ 7 & 8 & 9 \end{bmatrix}$$

**IMPORTANT:** *Notice the order in which the rows are entered. The second row listed is the row that gets replaced. If I wanted to put the result in the third row instead of the second, I would have entered the 2 before the 3.*

E:*row( Syntax: ∗row(*value,matrix,row*). Multiplies the given row in the specified matrix by *value*.

**Example 3.8:**

Assume the matrix $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$ is stored as [A].

To multiply the first row by −4, press MATRX ▶, select E:*row(, press (-) 4 , MATRX, select 1:[A], and press , 1 ) ENTER.

The result displayed on your screen should be

$$\begin{bmatrix} -8 & -12 & 4 \\ 3 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$$

F:*row+( Syntax: ∗row+(*value,matrix,rowA,rowB*). Multiplies *rowA* in the specified matrix by *value*, adds the result to *rowB*, and replaces *rowB* with the result.

**Example 3.9:**

Assume the matrix $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$ is stored as [A].

Suppose you want to add 9 times row one to row three and replace row three with the result.

Press MATRX ▶, select F:*row+(, press 9 , MATRX, select 1:[A], and press , 1 , 3 ) ENTER.

The result displayed on your screen should be

$$\begin{bmatrix} 2 & 3 & -1 \\ 3 & 50 & \\ 25 & 35 & 0 \end{bmatrix}$$

**Important note:** *None of the operations described in this section save the result. Thus, if you want to keep the result of your calculation, you need to save the result manually. If, for example, you wish to replace matrix* [A] *with the result of the last example, do the calculation as described above and then press* STO▶ MATRX, select 1:[A], and press ENTER.

## 3.4 Using the PIVOT Program

**Before you begin:**

- Get the PIVOT program from Appendix B of your text. Alternatively, you can probably get it from a classmate or from your instructor.
- Make sure that the matrix you want to work with is stored as matrix [A]. The PIVOT program can ONLY access this matrix.

**WARNING:** The PIVOT program changes matrix [A]! If you wish to keep this matrix, enter it as a different matrix (say, [B]), return to the home screen, and press MATRX, select 2:[B], press STO▶ MATRX, select 1:[A], and press ENTER. The matrix is now stored as both [A] and [B]. If you mess up while using the PIVOT program, you can restore matrix [A] to its original state by following the above keystrokes again.

Once you have the program and matrix [A] is ready, run the PIVOT program by pressing PRGM, selecting the program PIVOT (what number the selection is depends on how many programs you have, but it will probably be the first one for most of you), and pressing ENTER.

The screen shows you matrix [A] and waits. If the matrix is too wide for the screen, you can use ▶ and ◀ to see the whole thing at this point. When you are done looking at the matrix, press ENTER.

The calculator now asks you what you want to do.

1 TO PIVOT
2 TO DIVIDE
3 TO STOP

What the calculator does next depends on which option you choose. To choose an option, press the appropriate number (1, 2, or 3) and press ENTER.

**PIVOT** This option allows you to "pivot" on a particular entry. When you select PIVOT, the calculator asks for a row. Press the number of the row the desired pivot entry is in and press ENTER. The calculator then asks for a column. Press the number of the column the desired pivot entry is in and press ENTER. The calculator then pivots on the entry indicated, shows you the result, and waits. Press ENTER to return to the above menu.

**DIVIDE** This option allows you to divide an entire row by a number. When you select DIVIDE, the calculator says "DIVIDE ROW". Enter the number of the row you wish to divide. The calculator then says "BY". Enter the number you wish to divide the row by. The calculator divides the indicated row by the desired value, shows you the result, and waits. Press ENTER to return to the above menu.

**QUIT** This option quits the program.

**Example 3.10:**

Use the PIVOT program to reduce the matrix $\begin{bmatrix} 2 & 3 & -1 & 5 \\ 3 & 5 & 0 & 2 \\ 7 & 8 & 9 & 10 \end{bmatrix}$ to reduced row echelon form.

Press PRGM, select PIVOT, and press ENTER. The screen shows

```
prgmPIVOT
    [[2 3  -1  5 ]
     [3 5  0   2 ]
     [7 8  9   10]]
```

The dots in the upper right corner indicate that the calculator is paused, so it is necessary to press ENTER to continue. Do so.

We will start by pivoting on the first entry in row 1. Press 1 ENTER to select the PIVOT option. Enter "1" for both ROW and COLUMN. After pivoting, the calculator shows

```
2 TO DIVIDE
3 TO STOP 1
ROW 1
COLUMN 1
  [[2 3   -1 5   ]
   [0 1   3   -11]
   [0 -5  25  -15]]
```

Next, we pivot on row 2, column 2. Press [1] [ENTER] to select PIVOT. This time, enter "2" for both ROW and COLUMN. As usual, the calculator will show the results and wait. Press [ENTER].

Now pivot on row 3, column 3. The result should be

```
2 TO DIVIDE
3 TO STOP 1
ROW 3
COLUMN 3
[[80 0   0   820 …
 [0   40 0   -230…
 [0   0   40  -70 …
```

All pivoting is done. All that is left is to change the first nonzero entry in each row into a 1. We do this by using the DIVIDE option. Press [ENTER] to return to the menu. This time, select [2] for DIVIDE. Divide row 1 by 80. The result is

```
2 TO DIVIDE
3 TO STOP 2
DIVIDE ROW 1
BY 80
[[1  0  0   10.25…
 [0 40  0   -230 …
 [0  0  40  -70  …
```

Press [ENTER] to return to the menu. Finish by dividing row 2 by 40 and row 3 by 40. After dividing row 3 by 40, return to the menu and press [3] [ENTER] to quit. The final display shows

```
 [[1 0 0 10.25]
  [0 1 0 -5.75]
  [0 0 1 -1.75]]
1 TO PIVOT
2 TO DIVIDE
3 TO STOP 3
            Done
```

You may want to have the entries in fraction form instead. To do this, press [MATRX], select [1:[A]], press [MATH], select [1:▸Frac], and press [ENTER]. The new display is

```
2 TO DIVIDE
3 TO STOP 3
            Done
[A]▸Frac
 [[1 0 0 41/4 ]
  [0 1 0 -23/4]
  [0 0 1 -7/4 ]]
```

# Chapter 4

# Built-in Solvers

## 4.1 The Equation Solver

The Equation Solver can solve for any variable in an equation. To simplify matters, we will restrict our discussion to equations with only one variable.

The Equation Solver is accessed through the MATH menu. To bring it up, press MATH and select 0:Solver... . If this is the first time that you have used the Equation Solver, you will get the following display:

```
EQUATION SOLVER
eqn:0=
```

If you have used the Equation Solver before, then the display shows the previous equation and any solutions. To start over, move up to the equation and press CLEAR.

To solve an equation with the Equation Solver, you must first set the equation equal to 0. Once the equation is set equal to 0, enter it into the Equation Solver. As you push ENTER, two new lines appear: "X=" and "bound={-1E99,1...}".

The "bound" line refers to the range of values for the variable in which the solution(s) may lie. If the solution does not lie within this range, then you will probably not find it. The default range, {-1E99,1E99}, covers enough of the real line to handle most solutions.

The "X=" (or "T=" or "Y=", etc—depending on the variable in the equation) line is where the real fun occurs. Here, you must begin by guessing what a solution to the equation is and entering that guess. Once you have entered a guess, make sure the cursor is on the "X=" line and press ALPHA [SOLVE] to get a solution. ([SOLVE] is the alpha function of the ENTER key.)

The solution (if one is found) will be displayed in place of your guess, a box will appear to the left of "X=" to indicate that this is a value the calculator found, and a new line appears: "left-rt=0". This value SHOULD always be 0. If it isn't, try again with a different guess.

If you get the error message below, press [2] and try a new guess.

```
ERR:BAD GUESS
1:Quit
2:Goto
```

This message usually occurs because the guess didn't lie within the range defined by "bound".

If you get this error message:

```
ERR:NO SIGN CHNG
1:Quit
2:Goto
```

then there is probably no solution to the equation within the requested range. (Note the word "probably". Unfortunately, the Equation Solver is not perfect.)

Let's look at some examples.

**Example 4.1:**
To keep things simple, let's solve the equation $x^2 = 4$. The solutions ($-2$ and 2) are easy to find algebraically, so we can check our work easily enough.

First, we must set the equation equal to 0, so subtract 4 from each side to get $x^2 - 4 = 0$. Press [MATH] and select [0:Solver...]. Enter the equation by pressing [ALPHA] [X] [$x^2$] [$-$] [4] [ENTER].

To begin with, assume we don't know that 2 is a solution, but we know that there is a positive solution. Enter 40 as a guess and press [ALPHA] [SOLVE]. Note that the solution you get is X=1.9999999999.... The calculator didn't get 2, but got pretty darn close. Rounding errors are not uncommon when the guess isn't very close to the actual solution.

Now try guessing 5 instead. After entering 5 as a guess, press [ALPHA] [SOLVE]. This time we get the correct answer, X=2.

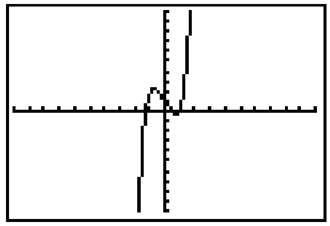
To get the negative solution, we need to enter a guess close to it. Try $-1.5$ as a guess and press [ALPHA] [SOLVE]. You should get X=-2.

Note from this example that, in order to get all solutions, you must know how many solutions there are and approximately what they are. But what if you don't know what the solutions are? After setting the equation equal to zero, draw a useful graph.

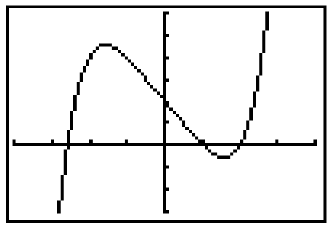
**Example 4.2:**
Use the Equation Solver to find all solutions to the equation $x^5 - 2x + 1 = 0$.

This equation is already set equal to 0, so I can go straight into solving—but I don't have any idea what to enter as a guess! I start by graphing $y = x^5 - 2x + 1$

to see how many $x$-intercepts there are and approximately where they occur. Looking at the graph on the standard window (X:[-10,10;1], Y:[-10,10;1]), I see that there appear to be three $x$-intercepts.



By changing the range to X:[-2,2;.5] and Y:[-1.5,3;.5], we see that the intercepts are at about $-1.25$, 0.5, and 1.



These are the values we will use as guesses in the Equation Solver.

Press MATH and select 0:Solver... . Clear out any previous equations and enter the new one. For the first guess, enter $-1.25$, then press ALPHA [SOLVE]. We find that the solution is $x \approx -1.2906488013467$ (it runs off the screen, so use the ▶ button to see it all. NOTE THAT THIS IS ONLY AN APPROXIMATE SOLUTION! THE CALCULATOR OFTEN CANNOT GIVE AN EXACT SOLUTION.

Press CLEAR, enter 0.5 as a guess, and press ALPHA [SOLVE]. The result should be $x \approx 0.51879006367591$.

Finally, press CLEAR, enter 1 as a guess, and press ALPHA [SOLVE]. The answer is $x \approx 1$. Note that I still said approximately. Until we test this solution in the original equation, we do not know if it's an exact answer or not. (It is, actually—try it.)

**Example 4.3:**
One very interesting thing that we should note is what happens if we guess 2 in the previous example.

Bring up the Equation Solver by pressing MATH and selecting 0:Solver... . If you did the previous example, the equation should already be entered, so put 2 in as a guess and press ALPHA [SOLVE]. The solution you get is $x \approx -1.2906488013467$. You might wonder why you didn't get 1, since that solution is closer. The answer lies in the procedure that the calculator uses to find the solution and is beyond the scope of this book. The point is that the solution you find is not always the one closest to your guess.

## 4.2 The TVM Solver

### What All the Variables Mean

First of all, "TVM" stands for "Time Value of Money". The TVM Solver is a tool which allows you to solve for payments on a loan, future value of an investment, term of a loan, and other quantities dealing with money over a long period of time.

To access the TVM Solver, press 2nd FINANCE and select 1:TVM Solver.... The first time you do this, you will get the following screen:

```
N=0
I%=0
PV=0
PMT=0
FV=0
P/Y=1
C/Y=1
PMT:END BEGIN
```

If you have used the TVM Solver before, then the values shown will be the ones entered the last time you used it. You need to know what all these variables mean.

**N** This stands for the number of payments made $(= \langle \text{P/Y} \rangle \cdot \langle \textit{number of years} \rangle)$.

**I%** This is the interest rate IN PERCENT. In other words, if the interest rate is 7.5%, enter "7.5" here, NOT "0.075".

**PV** This is the Principal Value (or original value) of the account/annuity. If the annuity is increasing, this value is negative. If the annuity is decreasing, this value is positive.

**PMT** This is the amount paid into the annuity at regular intervals. This means further deposits into an increasing annuity or payments on a loan. Either way, this value is negative.

**FV** This is the Final Value or Future Value of the annuity. It is how much the annuity is expected to be worth in the end. If you are paying off a loan, it is 0. Otherwise, its value depends on circumstances, but is generally positive.

**P/Y** This is the number of payments into the annuity (or on the loan) per year.

**C/Y** This is the number of times interest is computed each year. For every example in Math 111, this number is the same as P/Y.

**PMT:END BEGIN** This has to do with whether payments are made at the beginning or at the end of the compounding periods. For work in Math 111, leave this with END selected, as indicated above.

The table below shows how the calculator notation translates into the notation from the book.

| Calculator Notation | Book Notation |
|---|---|
| N | $mt$ |
| I% | $r \cdot 100$ |
| PV | $P$ |
| PMT | $W$ or $M$ |
| FV | $A$ |
| P/Y | $m$ |
| C/Y | $m$ |

If there is no variable in the formula in the book that corresponds to a particular calculator variable, enter 0 for that variable.

When entering variables into the TVM Solver, it is necessary to pay attention to what sign the number has. Any number which indicates a "cash outflow" (i.e., a decrease in the amount of money on hand) will be negative. Any number which indicates an increase in the amount of money on hand will be positive. Generally, receiving a loan or liquidating an investment that has reached maturity increases the amount of money on hand, whereas investing money or making payments on a loan decreases the amount of money on hand. Thus, loan amounts and future values will generally be entered as positive values, but initial investments and payments will be generally entered as negative values.

## Compound Interest

The compound interest formula is $A = P\left(1+\frac{r}{m}\right)^{mt}$. The table above shows how these variables translate into the TVM Solver variables. You use compound interest when the following two conditions are BOTH satisfied:

- Interest/dividends are computed based on the present value of the investment, and NOT on how much was originally invested.
- There are no further deposits or withdrawals being made. In other words, the investment is left alone and allowed to mature.

If the first condition is not satisfied, you are dealing with simple interest. If the second is not satisfied, then you are dealing with an annuity (either increasing or decreasing).

**Example 4.4:**
Suppose you deposit $1000 into a savings account earning 4% interest compounded monthly. If no other deposits or withdrawals are made, how long will it take for the balance to reach $1500?

Without the TVM Solver, this problem would require logarithms to solve properly, but the TVM Solver simplifies the process greatly. Before we can use the TVM Solver, however, we need to figure out what values go in place of all the variables.

The table below shows what the value of each TVM Solver variable is in this problem.

| Variable | Value |
|---|---|
| $\mathbb{N}$ | *unknown* |
| I% | 4 |
| PV | -1000 |
| PMT | 0 |
| FV | 1500 |
| P/Y | 12 |
| C/Y | 12 |

Press 2nd FINANCE and select 1:TVM Solver..., then enter these values in the appropriate places. PV is negative because it represents a "cash outflow".

**NOTES:** (1) *Don't worry about the entry for* $\mathbb{N}$. *Its value will be computed by the calculator, we we can ignore it for now.*
(2) *When you enter* 12 *for* P/Y, C/Y will automatically change.

Use ▲ to move back to the top line and press ALPHA SOLVE. The calculator computes $\mathbb{N} \approx 121.8421525$. THIS DOES NOT MEAN THAT IT TAKES 121.84 YEARS. Remember that $\mathbb{N} = mt$ and, in this case, $m = 12$. Thus, to find the amount of time, we must compute $t = \mathbb{N}/12$.

It is tempting at this point to return to the home screen and enter "121.8421525/12" to find $t$. While this works for most real-life cases, it is not entirely accurate. The calculator "knows" more decimal places than it shows. Thus, to get the most accurate calculation, we need to use what the calculator knows instead of what the calculator shows. To get $\mathbb{N}$, press 2nd FINANCE ▶ and select 1:$\mathbb{N}$. Thus, after returning to the home screen, the keystrokes to find $t$ are: 2nd FINANCE ▶, select 1:$\mathbb{N}$, press ÷ 1 2 ENTER. This gives an answer of $t \approx 10.15351271$. The answer, then, is that it will take approximately 10.15 years (or about 10 years and 2 months) for the balance to reach $1500.

**Example 4.5:**
An investment grows at a steady rate of 8.5% per year for 5 years. If the end value of the investment is $4999.66, how much was invested initially?

The table below shows the value of the TVM Solver variables:

| Variable | Value |
|---|---|
| $\mathbb{N}$ | 5 |
| I% | 8.5 |
| PV | *unknown* |
| PMT | 0 |
| FV | 4999.66 |
| P/Y | 1 |
| C/Y | 1 |

Enter these values, move to the PV line and press ALPHA [SOLVE]. You should get the result PV $\approx -3325.001001$. Thus, we conclude that the initial investment was $3325.00 (the rest of the decimal places make no sense in our monetary system, so we ALWAYS round to the nearest cent).

If an investment is losing money, the same work can be done with a negative interest rate.

**Example 4.6:**
You invest $1000 in your brother's company, Really Bad Investment, Inc. For the next 10 years, the stock decreases in value by 10% each year. What is the value of your investment at the end of those 10 years.

The TVM Solver variables are:

| Variable | Value |
|---|---|
| N | 10 |
| I% | −10 |
| PV | −1000 |
| PMT | 0 |
| FV | *unknown* |
| P/Y | 1 |
| C/Y | 1 |

Enter these values, move to the FV line, and press ALPHA [SOLVE]. You should get that FV $\approx 348.6784401$, meaning that the value of the stock after 10 years is $348.68.

## Increasing Annuities

The increasing annuity formula is $A = M\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}$. The table on page 36 shows how these variables translate into the TVM Solver variables. You are dealing with an increasing annuity when the following conditions are ALL satisfied:

- The value of the annuity is increasing.
- Interest/dividends/finance charges are computed using the current value of the annuity.
- Regular payments are made into the annuity which help to increase its value.
- No withdrawals are made against the annuity.

A retirement account (prior to retirement) is an excellent example of an increasing annuity.

**Example 4.7:**
This is example 1 on page 581 of the Math 111 text.

Every month for 10 years, you deposit $100 into a retirement account that pays 5% interest per year compounded monthly. How much money will there be in the account at the end of those 10 years?

The values of the TVM Solver variables are shown below:

| Variable | Value |
|---|---|
| N | 120 |
| I% | 5 |
| PV | 0 |
| PMT | $-100$ |
| FV | *unknown* |
| P/Y | 12 |
| C/Y | 12 |

Enter these values, move to FV, and press ALPHA [SOLVE]. The result you should get is FV $\approx 15,528.22794$, meaning that the end value of the account is $15,528.23.

## Decreasing Annuities and Loans

The decreasing annuity (or loan) formula is $P = W\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}$. The table on page 36 shows how these variables translate into the TVM Solver variables. You are dealing with a decreasing annuity or loan when the following conditions are ALL satisfied:

- The value of the annuity/amount owed is decreasing.
- Interest/finance charges are computed using the current value of the annuity/amount owed.
- Regular payments are made against the annuity/loan which help to decrease its value.
- No transactions (besides interest/finance charges) occur which would increase the value of the annuity.

A retirement account (after retirement) and a mortgage are excellent examples of decreasing annuities.

**Example 4.8:**
This is example 9 from page 590 of the Math 111 text.

*Discover Card* is charging an annual rate of 14.9% interest on outstanding balances (as quoted on *Discover Card*'s billing statements in April 1994). You have an outstanding balance of $500 and calculate that you can afford to make payments of $20 per month. How long will it take you to settle the debt?

The book, at this point, goes into how to use graphs to solve this problem. As the proud owner of a TI-83, however, you can do this problem simply by using the TVM Solver.

The values of all the TVM Solver variables are indicated in the table below:

| Variable | Value |
|---|---|
| N | *unknown* |
| I% | 14.9 |
| PV | 500 |
| PMT | −20 |
| FV | 0 |
| P/Y | 12 |
| C/Y | 12 |

**Note:** PV *is positive because the use of the credit card has, in effect, increased the amount of money on hand by $500.*

Enter these values, move to the top line, and press ALPHA [SOLVE]. The result is $\mathbb{N} \approx 30.11842309$. THIS DOES NOT MEAN THAT IT TAKES OVER 30 YEARS. Remember, $\mathbb{N} = mt$ and $m = 12$ in this case. Thus, $t = \frac{\mathbb{N}}{12}$.

To compute this, return to the home screen, press 2nd FINANCE ▶, select 1:N, and press ÷ 1 2 ENTER. The answer is $t \approx 2.50986859$, indicating that it will take about $2\frac{1}{2}$ years to pay off the debt.

**NOTE:** *It is important in the above example that you not use the credit card in the meantime. If the card gets used, then the transaction would increase the amount of the debt and thus violate one of the properties of a decreasing annuity/loan.*

# Appendix A

# Using the TI-83 Plus

## A.1 What is a TI-83 Plus?

The basic difference between the TI-83 and the TI-83 Plus is one of upgradeability. The TI-83 Plus is designed with upgrading in mind. As new applications for the TI-83 Plus are designed, they will be posted to a special Web site from which they can be downloaded and, with the appropriate software and cable, installed on the TI-83 Plus. Once installed, these applications can be accessed by using the APPS button, which the TI-83 (no Plus) doesn't have. Because of the addition of this button, some things needed to be moved around.

## A.2 Finding the TVM Solver

There is no 2nd FINANCE sequence on the TI-83 Plus, so it won't do you any good to hit that sequence, as described in section 4.2. The TVM Solver is considered an application (of the type that can be downloaded and installed), so is found by hitting APPS 1:Finance... 1:TVM Solver....

## A.3 Finding the MATRX Button

The APPS button on the TI-83 Plus is in the same location as the MATRX button on the TI-83. For that reason, access to the matrix names list and editor had to be changed. On the TI-83 Plus, MATRX is the second function of the $x^{-1}$ button, so press 2nd MATRX. All other operations are the same.

# Index