MATH-141 Precalculus Exam 4 Practice

NOTE: Show your work and circle or highlight your final answer.

1. Simplify:
   a. \( (3yz^{-4})^{\frac{1}{3}} (7y^{-3}z^5)^{-3} \)
   \[
   \frac{3^{\frac{1}{3}} y^{\frac{1}{3}} z^{\frac{1}{4}}}{3^{\frac{1}{3}} y^{\frac{1}{3}} z^{\frac{1}{4}}} = \frac{2}{3^{\frac{1}{3}}} \frac{y^{\frac{1}{3}}}{y^{\frac{1}{3}}} z^{\frac{1}{4}} z^{\frac{1}{4}} = \frac{2}{3^{\frac{1}{3}}} \frac{1}{3^{\frac{1}{3}}} = \frac{2}{3^{\frac{2}{3}}} = \frac{2}{\sqrt[3]{9}}
   \]
   b. \( \left( \frac{8a^2 c^{-2}}{10a^4 c^3} \right)^{\frac{3}{2}} \left( \frac{25b^{-4}c^{-5}}{2b^3c^{-4}} \right)^4 \)
   \[
   = \frac{8^{\frac{3}{2}}}{10^{\frac{3}{2}}} \frac{a^{-3}}{a^{6}} c^{-3} \frac{25^{\frac{4}{4}}}{2^{\frac{12}{4}}} b^{-12} c^{-20} = \frac{531}{10} \frac{1}{a^3} \frac{1}{b^{12}} \frac{1}{c^{20}}
   \]

2. Find the domain of the following functions. Do not use the graphing calculator.
   a. \( f(x) = \sqrt{7x - 5} \)
   \( 7x - 5 \geq 0 \)
   \( x \geq \frac{5}{7} \)
   \( \left[ \frac{5}{7}, \infty \right) \)
   b. \( f(x) = \frac{13}{3x^2 - 7x - 10} \)
   \( 3x^2 - 7x - 10 \neq 0 \)
   \( (3x - 10)(x + 1) \neq 0 \)
   \( x \neq \frac{10}{3}, -1 \)

3. The Golden Gate Bridge is supported by two huge cables strung between the towers at each end of the bridge. The function \( f(x) = .001543x^2 - 7.2521x + 526.112 \) gives the approximate height of the cables above the roadway at a point on the road \( x \) feet from one of the towers. The cables touch the road halfway between the two towers. How far apart are the towers?

4. A rectangular garden next to a building is to be fenced on three sides. Fencing for the side parallel to the building costs $80 per foot, and material for the other two sides costs $20 per foot. If $3,600 is to be spent on fencing, what are the dimensions of the garden with the largest possible area?
5. Graph the following. Do not use the graphing calculator.
   a. \( y = -3(x - 4)^2 - 8 \)
      
      \[ \text{opens: } \downarrow \]
      
      \[ \text{vertex: } (4, -8) \]
      
      \[ \text{axis: } x = 4 \]
      
      \[ y = -3(x - 4)^2 - 8 \]
      
      \[ y = 0 \Rightarrow -3(x - 4)^2 - 8 = 0 \]
      
      \[ 3(x - 4)^2 = -8 \]
      
      \[ x - 4 = \pm \sqrt{\frac{-8}{3}} \]
      
      \[ x = 4 \pm \sqrt{\frac{8}{3}} \]
      
      \[ y = -3(x - 4)^2 - 8 \]
   b. \( y = 3x^2 + 6x \)
      
      \[ \text{opens: } \uparrow \]
      
      \[ \text{vertex: } (-1, -3) \]
      
      \[ \text{axis: } x = -1 \]
      
      \[ y = 3(x + 1)^2 - 3 \]
      
      \[ y = 0 \Rightarrow 3(x + 1)^2 - 3 = 0 \]
      
      \[ 3(x + 1)^2 = 3 \]
      
      \[ x + 1 = \pm \sqrt{1} \]
      
      \[ x = -1 \pm 1 \]
      
      \[ x = 0, -2 \]

6. Give the domain of each function as well as all asymptotes. Do not use the graphing calculator.
   a. \( f(x) = \frac{x + 9}{x^2 - 9} \)
      
      \[ \text{Domain: } (-\infty, -3) \cup (-3, 3) \cup (3, \infty) \]
      
      \[ \text{VA: } x = 3, -3 \]
      
      \[ \text{HA: } y = 0 \]
   b. \( f(x) = \frac{7x^2 + x}{8x^2 - 2x - 3} \)
      
      \[ \text{Domain: } (-\infty, -\frac{1}{4}) \cup (-\frac{1}{4}, \frac{3}{8}) \cup (\frac{3}{8}, \infty) \]
      
      \[ \text{VA: } x = -\frac{1}{4}, \frac{3}{8} \]
      
      \[ \text{HA: } y = \frac{7}{8} \]

7. Write an equation with the given roots.
   a. Roots: 0, 2, -7
      
      \( y = (x - 0)(x - 2)(x + 7) \)
      
      \[ y = x(x - 2)(x + 7) \]
      
      \[ y = x^3 + 5x^2 - 14x \]
   b. Roots: 3, 3, 2-i
      
      \( y = (x - 3)^2(x - 2 + i)(x - 2 - i) \)
      
      \[ y = ((x - 3)^2)((x - 2 + i)(x - 2 - i)) \]
      
      \[ y = x^4 - 10x^3 + 38x^2 - 66x + 45 \]

8. Use long division.
   a. \( \frac{x^5 - x^3 + x - 2}{x - 3} \)
      
      \[ x^4 + 3x^3 + 8x^2 + 24x + 73 + \frac{217}{x - 3} \]
      
      \[ x^3 + 0x^2 - x + 2 \]
      
      \[ -x^3 + 3x^2 - x + 2 \]
      
      \[ \frac{3x^3 + 8x^2 + 24x + 73}{x - 3} \]
      
      \[ -3x^3 + 9x^2 + 6x + 2 \]
      
      \[ x^2 + 2x + 1 \]
      
      \[ \frac{73x - 2}{x^2 + 2x + 1} \]
      
      \[ -73x + 2 \]
      
      \[ -x^2 + 5x - 1 \]
      
      \[ \frac{2x^2 + 5x + 1}{15} \]
   b. \( \frac{x^3 - 4x^2 - 4x + 16}{x^2 - 5x + 1} \)
      
      \[ x^2 + \frac{15}{x^2 - 5x + 1} \]
      
      \[ -x^2 + 5x - 1 \]
      
      \[ \frac{2x^2 + 5x + 1}{15} \]
      
      \[ -2x^2 + 10x - 1 \]
      
      \[ -2x^2 + 2x - 1 \]
      
      \[ \frac{18x + 15}{15} \]

9. Use the Factor Theorem to determine whether or not \( h(x) \) is a factor of \( f(x) \).
   a. \( h(x) = x + 1, \ f(x) = x^3 + 5x^2 + x^2 + 4x \)
      
      \[ x = -1 \]
      
      \[ (-1)^3 + 5(-1)^2 + (-1)^2 + 4(-1) \]
      
      \[ -1 - 5 + 1 - 4 = -9 \]
      
      \[ \boxed{\text{No}} \]
   b. \( h(x) = x - 2, \ f(x) = x^3 - 5x^2 + 13x - 14 \)
      
      \[ x = 2 \]
      
      \[ (2)^3 - 5(2)^2 + 13(2) - 14 \]
      
      \[ 8 - 20 + 26 - 14 \]
      
      \[ \boxed{0} \]
10. Determine whether the given graph could possibly be the graph of a polynomial function. If so, what degree is the polynomial likely to be?

a. [Graph with a degree indication: yes, degree at least 5]

b. [Graph with a no indication]

11. Find the remainder when $f(x) = x^3 + 4x^2 - 10x - 8$ is divided by $g(x) = x + 10$, without using division.

\[x = -10\]

\[(-10)^3 + 4(-10)^2 - 10(-10) - 8\]

\[= -1000 + 400 + 100 - 8\]

\[= -508\]